

# CBSE 2019

## MATHEMATICS

(SET-1)

### (Solutions)

1.  $(x+5)^2 = 2(5x-3)$

$$\Rightarrow x^2 + 25 + 10x = 10x - 6$$

$$\Rightarrow x^2 + 31 = 0$$

$$\Rightarrow 1x^2 + 0x + 31 = 0$$

$$a = 1, b = 0, c = 31$$

$$\text{Discriminant} = b^2 - 4ac$$

$$\Rightarrow 0^2 - 4(1)(31)$$

$$\Rightarrow -124$$

2.  $\frac{27}{2^3 5^4 3^2}$

$$= \frac{3}{2^3 5^4} \times \frac{2}{2}$$

$$= \frac{6}{(2 \times 5)^4}$$

$$= \frac{6}{10^4}$$

$\therefore$  Number will terminate after 4 decimal places

OR

3	429
11	143
13	13
	1

$$429 = 3 \times 11 \times 13$$

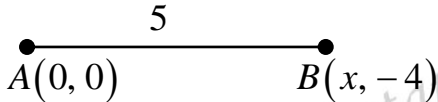
3. The list of first 10 multiples of 6 is

6, 12, 18,....., 60

which forms an AP

$$S_{10} = \frac{10}{2}[6 + 60]$$

$$S_n = 330$$

4. 

$$AB = \sqrt{(x-0)^2 + (-4-0)^2} \quad (\text{Using Distance formula})$$

$$\Rightarrow 5 = \sqrt{x^2 + 16}$$

$$\Rightarrow 25 = x^2 + 16 \quad (\text{Squaring both sides})$$

$$\Rightarrow x^2 = 9$$

$$\Rightarrow x = \pm 3$$

5.  $OC \perp AB$  (AB is tangent to the smaller circle)

In  $\triangle OBC$

$$a^2 = b^2 + CB^2$$

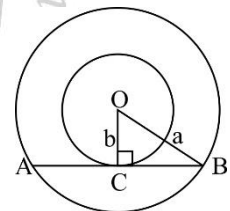
$$CB^2 = a^2 - b^2$$

$$CB = \sqrt{a^2 - b^2}$$

$$AB = 2CB$$

(Perpendicular from the centre bisects the chord)

$$\therefore AB = 2\sqrt{a^2 - b^2}$$



6. In  $\triangle PQS$

$$PQ^2 = 3^2 + 4^2 \quad (\text{By Pythagoras theorem})$$

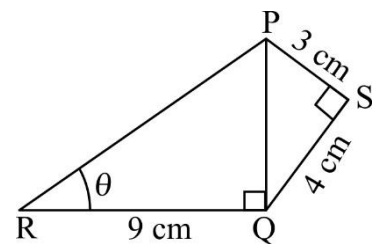
$$PQ^2 = 25$$

$$PQ = 5 \text{ cm}$$

In  $\triangle PQR$

$$\tan \theta = \frac{PQ}{RQ}$$

$$\tan \theta = \frac{5}{9}$$



OR

$$\tan \alpha = \frac{5}{12}$$

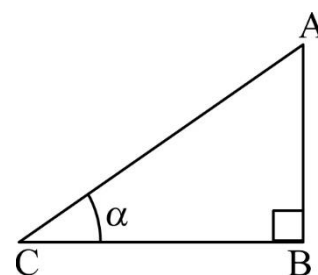
let  $AB = 5k$  and  $BC = 12k$

$$AC^2 = (5k)^2 + (12k)^2$$

$$\Rightarrow AC^2 = 169k^2$$

$$\Rightarrow AC = 13k$$

$$\sec \alpha = \frac{AC}{BC} = \frac{13}{12}$$



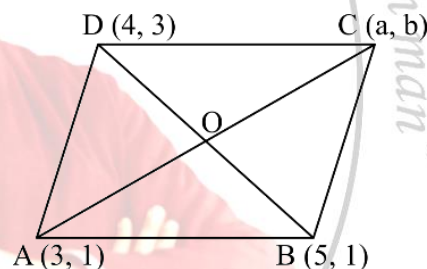
7. Mid point of  $AC =$  Mid point of  $BD$

(Diagonals of a parallelogram bisect each other)

$$\left( \frac{3+a}{2}, \frac{1+b}{2} \right) = \left( \frac{5+4}{2}, \frac{1+3}{2} \right) \quad (\text{By mid point formula})$$

$$\frac{3+a}{2} = \frac{9}{2} \quad \text{and} \quad \frac{1+b}{2} = 2$$

$$\therefore a = 6 \quad \text{and} \quad b = 3$$



OR

P divides AB in the ratio 1:2

$$\text{Coordinates of } P = \left( \frac{1 \times 0 + 2(-2)}{1+2}, \frac{1 \times 8 + 2 \times 0}{1+2} \right)$$

$$\text{Coordinates of } P = \left( \frac{-4}{3}, \frac{8}{3} \right)$$

(By section formula)

Q divides AB in the ratio 2:1

$$\text{Coordinates of } Q = \left( \frac{2 \times 0 + 1 \times (-2)}{2+1}, \frac{2 \times 8 + 1 \times (0)}{2+1} \right)$$

$$\text{Coordinates of } Q = \left( \frac{-2}{3}, \frac{16}{3} \right)$$



8.  $3x - 5y = 4$  .....(i)

$9x - 2y = 7$  .....(ii)

Multiply (i) by 3

$3(3x - 5y = 4)$  .....(iii)

Solving (iii) and (ii)

$9x - 15y = 12$

$9x - 2y = 7$

$$\begin{array}{r} - \quad + \quad - \\ \hline \end{array}$$

$-13y = 5$

$y = \frac{-5}{13}$

Substituting this value of y in (i) we

get

$x = \frac{9}{13}$

9. Using Euclid's division algorithm to find HCF of 65 and 117

$117 = 65 \times 1 + 52$  .....(i)

$65 = 52 \times 1 + 13$  .....(ii)

$52 = \boxed{13} \times 4 + 0$  .....(iii)

HCF = 13

from (ii)

$65 - 52 = 13$  ... (iii)

from (i)

$52 = 117 - 65$  put in (iii)

$65 - (117 - 65) = 13$

$65 \times 2 - 117 = 13$  ... (iv)

on comparing (iv) with  $65n - 117$

we get  $n = 2$

**OR**

Minimum distance each should walk so that each can cover the same distance in complete steps is LCM of 30, 36 and 40.

$$30 = 2 \times 3 \times 5$$

$$36 = 2^2 \times 3^2$$

$$40 = 2^3 \times 5$$

$$\begin{aligned} \text{LCM}(30, 36, 40) &= 2^3 \times 3^2 \times 5 \\ &= 360 \end{aligned}$$

$\therefore$  Minimum distance = 360 cm

10. Possible outcomes = {1, 2, 3, 4, 5, 6}

Probability of an event =  $\frac{\text{Number of outcomes favourable to event}}{\text{Total number of equally likely outcomes}}$

(i) Favourable outcomes = {4, 6}

$$P(\text{a composite number}) = \frac{2}{6}$$

(ii) Favourable outcomes = {2, 3, 5}

$$P(\text{a prime number}) = \frac{3}{6}$$

11.  $x^2 - 8x + 18 = 0$

$$\Rightarrow (x)^2 - 2x(4) + 4^2 - 4^2 + 18 = 0$$

$$\Rightarrow (x-4)^2 + 2 = 0$$

$$\Rightarrow (x-4)^2 = -2$$

Since square of a number cannot be negative, therefore this equation has no solution.

12. Possible outcomes = {7, 8, 9, 10,....., 40}

$$\begin{aligned}\therefore \text{Total outcomes} &= 40 - 7 + 1 \\ &= 34\end{aligned}$$

Probability of an event =  $\frac{\text{Number of outcomes favourable to event}}{\text{Total number of equally libely outcomes}}$

Favourable outcomes = {7, 14, 21, 28, 35}

$$P(\text{multiple of 7}) = \frac{5}{34}$$

13. Given :  $AD \perp BC$

$$BD = 3CD$$

To prove :  $2AB^2 = 2AC^2 + BC^2$

Proof : let  $CD = x$ ,  $BD = 3x$

$$\therefore BC = 4x$$

In  $\triangle ABD$

$$AB^2 = AD^2 + BD^2$$

$$2AB^2 = 2(AD^2 + BD^2) \quad (\text{Multiply both sides by 2})$$

$$= 2(AD^2 + (3x)^2)$$

$$2AD^2 + 18x^2$$

$$2(AC^2 - CD^2) + 18x^2 \quad (AC^2 = AD^2 + DC^2)$$

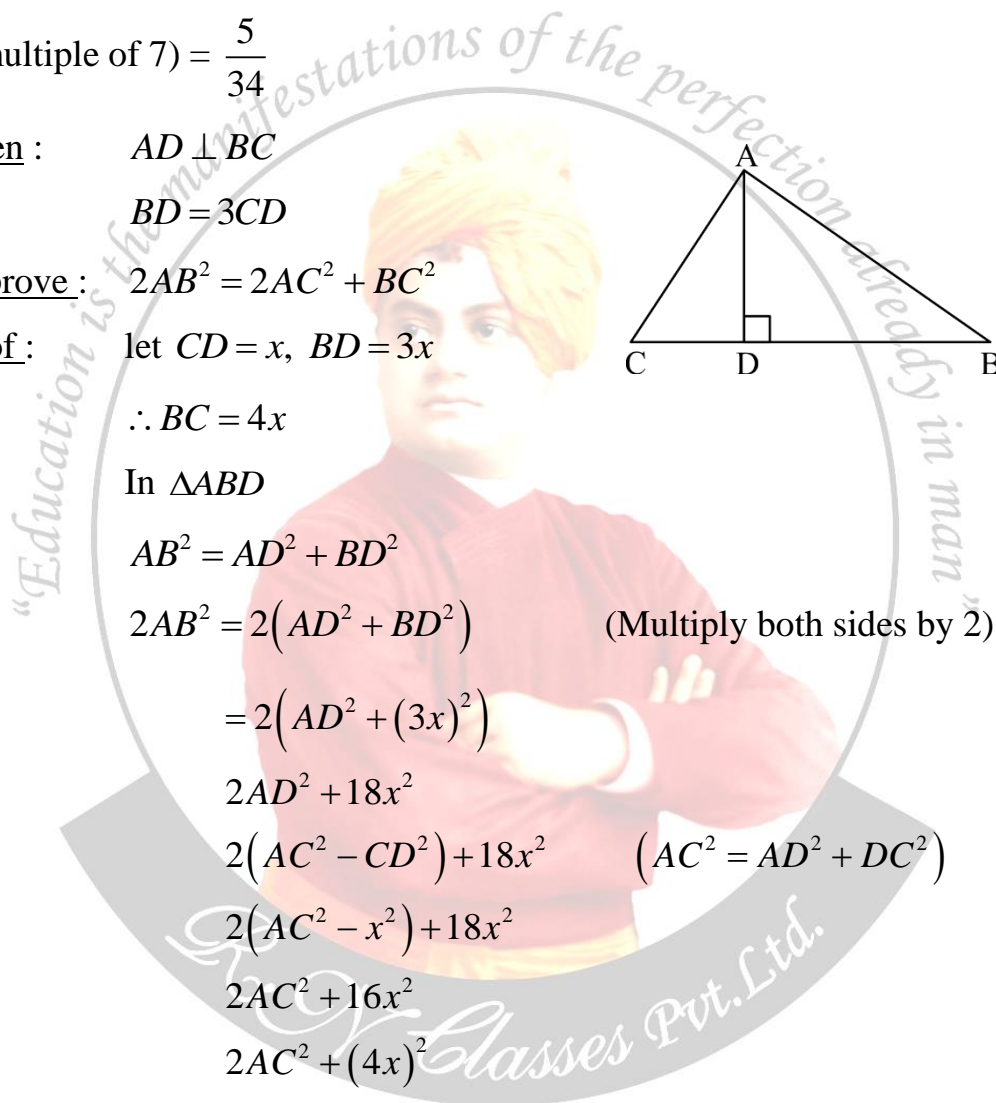
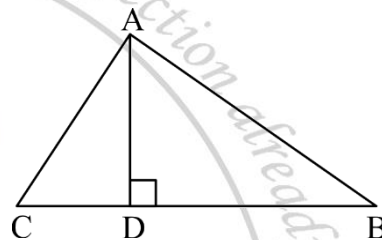
$$2(AC^2 - x^2) + 18x^2$$

$$2AC^2 + 16x^2$$

$$2AC^2 + (4x)^2$$

$$2AC^2 + BC^2$$

Hence proved

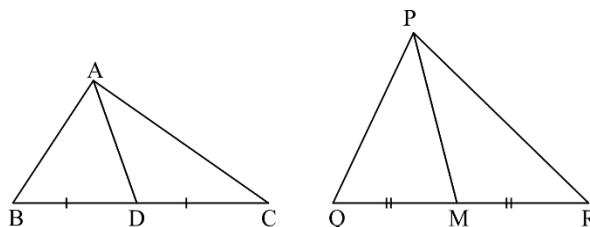




OR

Given :  $\triangle ABC \sim \triangle PQR$

$AD$  and  $PM$  are medians



To prove :  $\frac{AB}{PQ} = \frac{AD}{PM}$

Proof :  $\angle B = \angle Q$  ( $\triangle ABC \sim \triangle PQR$ ) .....(i)

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$$

$$\frac{AB}{PQ} = \frac{BD}{QM} \quad \text{.....(ii)}$$

from (i) & (ii)

$\triangle ABD \sim \triangle PQM$  (SAS criteria)

$$\therefore \frac{AB}{PQ} = \frac{AD}{PM}$$

14.  $(x^3 - 3x + 1) \overline{) x^5 - 4x^3 + x^2 + 3x + 1} (x^2 - 1$

$$\begin{array}{r} x^5 - 3x^3 + x^2 \\ - \quad + \quad - \\ \hline \end{array}$$

$$-x^3 + 3x + 1$$

$$-x^3 + 3x - 1$$

$$\begin{array}{r} + \quad - \quad + \\ \hline \end{array}$$

$$\underline{\quad 2}$$

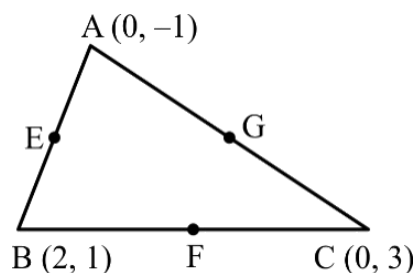
As remainder is 2,  $\therefore g(x)$  is not the factor of  $p(x)$

15. Let  $D, E$  and  $F$  are mid points of  $AB, BC$  and  $AC$  respectively.

$$D\left(\frac{0+2}{2}, \frac{-1+1}{2}\right) = D(1, 0)$$

$$E\left(\frac{0+0}{2}, \frac{-1+3}{2}\right) = E(0, 1)$$

$$F\left(\frac{2+0}{2}, \frac{1+3}{2}\right) = F(1, 2)$$



$$\text{Area of } \triangle DEF = \text{Numerical value of } \frac{1}{2}[1(1-2)+0(2-0)+1(0-1)]$$

$$= \text{Numerical value of } \frac{1}{2}[-2]$$

$$= \text{Numerical value of } -1$$

$$\therefore \text{Area of } \triangle DEF = 1 \text{ sq units.}$$

16.  $x - y = -1$

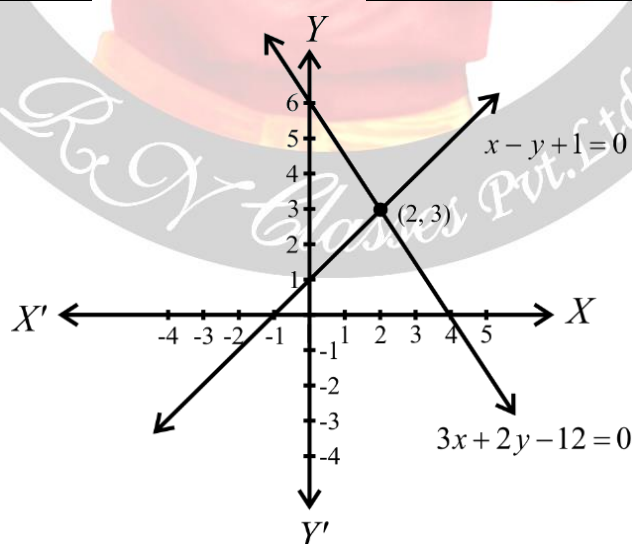
$$3x + 2y = 12$$

$$x = y - 1$$

$$x = \frac{12 - 2y}{3}$$

$x$	-1	0	1
$y$	0	1	2

$x$	0	6	2
$y$	6	-3	3



$$x = 2, \quad y = 3$$



17. Let us assume that  $\sqrt{3}$  is a rational number

$$\sqrt{3} = \frac{p}{q} \quad (p, q \text{ are coprime})$$

and  $q \neq 0$

Squaring both sides

$$3 = \frac{p^2}{q^2}$$

$$3q^2 = p^2 \dots\dots\dots(i)$$

3 divides  $p^2$

3 divides  $p$

$$\therefore p = 3m$$

Putting this value in (i)

$$3q^2 = (3m)^2$$

$$q^2 = 3m^2$$

3 divides  $q^2$

3 divides  $q$

Therefore  $p$  and  $q$  have 3 as a common factor. But this contradicts our assumption that  $p$  and  $q$  are co prime.

This contradiction has arisen due to our wrong assumption that  $\sqrt{3}$  is rational  
Therefore  $\sqrt{3}$  is an irrational number.

**OR**

Let the largest number be ' $a$ ' according to Euclid's division lemma

$$1251 = aq + 1$$

$$\Rightarrow aq = 1250 \quad \dots\dots\dots(i)$$

$$9377 = aq' + 2$$

$$\Rightarrow aq' = 9375 \quad \dots\dots\dots(ii)$$

$$15628 = aq'' + 3$$

$$\Rightarrow aq'' = 15625 \quad \dots\dots(iii)$$

from (i), (ii) and (iii)

$$a = \text{HCF} (1250, 9375, 15625)$$

$$1250 = 2 \times 5^4$$

$$9375 = 3 \times 5^5$$

$$15625 = 5^6$$

$$\text{HCF} (1250, 9375, 15625) = 5^4$$

$$\therefore a = 625$$

18. Given:

$$(i) \quad A + B + C = 180^\circ$$

$$\text{To prove: } \sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

$$\text{Proof: LHS: } \sin\left(\frac{B+C}{2}\right)$$

$$= \sin\left(\frac{180-A}{2}\right)$$

$$= \sin\left(90 - \frac{A}{2}\right)$$

$$= \cos\frac{A}{2}$$

$$= \text{RHS}$$

Hence Proved

$$(ii) \quad A = 90$$

$$B + C = 180 - 90$$

$$B + C = 90$$

$$\tan\left(\frac{B+C}{2}\right) = \tan\left(\frac{90}{2}\right)$$

$$= \tan 45$$

$$= 1$$

OR

$$\tan(A + B) = 1$$

$$\Rightarrow A + B = 45^\circ \dots\dots\dots (i)$$

$$\tan(A - B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow A - B = 30^\circ \dots\dots\dots (ii)$$

Adding (i) & (ii)

$$2A = 75^\circ$$

$$\Rightarrow A = \left(\frac{75}{2}\right)^\circ$$

Putting this value in (i)

$$\text{we get } B = \left(\frac{15}{2}\right)^\circ$$

19. Join OT. Let it intersect PQ at the point R. Then  $\triangle TPQ$  is isosceles and TO is the angle bisector of  $\angle PTQ$ .

So,  $OT \perp PQ$  and therefore, OT bisects  $PQ \therefore PR = RQ = 4\text{cm}$ .

$$\text{Also, } OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2} \text{ cm} = 3\text{cm}$$

$$\text{Now, } \angle TPR + \angle RPO = 90^\circ = \angle TPR + \angle PTR$$

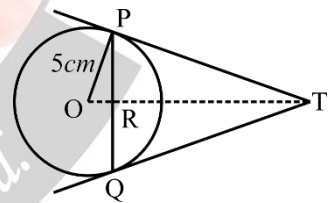
$$\text{So, } \angle RPO = \angle PTR.$$

$$\therefore \triangle TRP \sim \triangle PRO \quad (\text{by AA similarity})$$

$$\Rightarrow \frac{TP}{PO} = \frac{RP}{RO}$$

$$\text{i.e. } \frac{TP}{5} = \frac{4}{3}$$

$$\Rightarrow TP = \frac{20}{3} \text{ cm.}$$



OR

Given: ABCD is circumscribing a circle

$\therefore AB, BC, CD, AD$  are tangents

To prove:  $\angle AOB + \angle COD = 180^\circ$  and  $\angle BOC + \angle AOD = 180^\circ$

Proof: Join  $OH, OG, OF, OE$

In  $\triangle AOH$  and  $\triangle AOG$

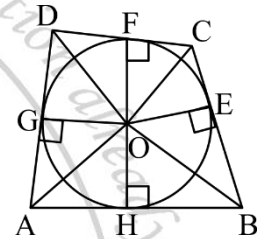
$\angle AGO = \angle AHO$  (tangent is perpendicular to the radius at the point of contact)

$OG = OH$  (radii)

$AO = AO$  (Common)

$\therefore \triangle AOH \cong \triangle AOG$  (RHS)

$\therefore \angle AOH = \angle AOG = x$  (CPCT)



Similarly,  $\angle BOH = \angle BOE = y$  ( $\triangle BOH \cong \triangle BOE$ )

$\angle COE = \angle COF = p$  ( $\triangle COE \cong \triangle COF$ )

$\angle FOD = \angle GOD = q$  ( $\triangle FOD \cong \triangle GOD$ )

$x + x + y + y + p + p + q + q = 360^\circ$  (angle at O)

$\Rightarrow 2(x + y + p + q) = 360^\circ$

$\Rightarrow x + y + p + q = 180^\circ$

$\Rightarrow \angle AOB + \angle COD = 180^\circ$   $\left( \begin{array}{l} \angle AOB = x + y \\ \angle COD = p + q \end{array} \right)$

Similarly, it can be proved that  $\angle BOC + \angle AOD = 180^\circ$

Hence Proved

20. Volume of water flowing through the canal in 30 minutes = Area irrigated  $\times$  height

Volume of water flowing through canal in 1 hour =  $10000 \times 6 \times 1.5$

$$\text{Volume of water flowing through canal in 30 minutes} = \frac{10000 \times 6 \times 1.5}{2}$$

Now,

$$\frac{10000 \times 6 \times 1.5}{2} = \text{Area irrigated} \times \frac{8}{100}$$

$$\therefore \text{Area irrigated} = 562500 \text{ m}^2$$

21.

Number of days	Number of students ( $f_i$ )	$x_i$	$f_i x_i$
0-6	10	3	30
6-12	11	9	99
12-18	7	15	105
18-24	4	21	84
24-30	4	27	108
30-36	3	33	99
36-42	1	39	39
	40		564

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{564}{40}$$

$$= 14.1$$

$\therefore$  Mean number of days is 14.1

22. Total area cleaned =  $2 \times (\text{Area cleaned by 1 wiper})$

$$\begin{aligned} &= 2 \times \left( \frac{\theta}{360} \times \pi r^2 \right) \\ &= 2 \times \left( \frac{120}{360} \times \frac{22}{7} \times (21)^2 \right) \\ &= 924 \text{ cm}^2 \end{aligned}$$

23. Let P be the required location of the pole.

Let  $BP = xm$

So,  $AP = (x+7)m$

Now,  $AB = 13m$  and since AB is a diameter,

$$\angle APB = 90^\circ \quad (\text{Angle in semicircle})$$

Therefore,  $AP^2 + PB^2 = AB^2$  (By Pythagoras theorem)

$$\text{i.e. } (x+7)^2 + x^2 = 13^2$$

$$\text{i.e. } x^2 + 14x + 49 + x^2 = 169$$

$$\text{i.e. } 2x^2 + 14x - 120 = 0$$

So, the distance 'x' of the pole from gate B satisfies the equation

$$x^2 + 7x - 60 = 0$$

So, it would be possible to place the pole if this equation has real roots. To see if this is so or not, let us consider its discriminant.

The discriminant is

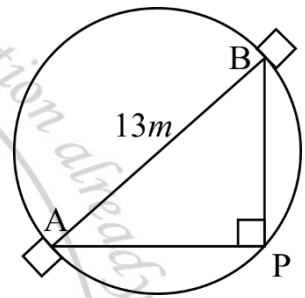
$$b^2 - 4ac = 7^2 - 4 \times 1 \times (-60) = 289 > 0$$

So, the given quadratic equation has two real roots, and it is possible to erect the pole on the boundary of the park.

$$x^2 + 7x - 60 = 0$$

By the quadratic formula,

$$x = \frac{-7 \pm \sqrt{289}}{2} = \frac{-7 \pm 17}{2}$$





Therefore,  $x = 5$  or  $-12$  (Rejected)

Thus, the pole has to be erected on the boundary of the park at a distance of 5m from the gate B and 12 m from the gate A.

24. Let first term be 'a' and common difference be 'd'

$$a_m = a + (m-1)d$$

$$a_n = a + (n-1)d$$

According to question

$$ma_m = na_n$$

$$\Rightarrow m(a + (m-1)d) = n(a + (n-1)d)$$

$$\Rightarrow ma + (m-1)md - na - (n-1)nd = 0$$

$$\Rightarrow a(m-n) + d((m-1)m - (n-1)n) = 0$$

$$\Rightarrow a(m-n) + d(m^2 - m - n^2 + n) = 0$$

$$\Rightarrow a(m-n) + d(m^2 - n^2 - (m-n)) = 0$$

$$\Rightarrow a(m-n) + d((m-n)(m+n) - (m-n)) = 0$$

$$\Rightarrow a(m-n) + d(m-n)(m+n-1) = 0$$

$$\Rightarrow (m-n)(a + (m+n-1)d) = 0$$

$$m-n=0 \quad \text{or} \quad a + (m+n-1)d = 0$$

$$m=n \quad \quad \quad a + (m+n-1)d = 0$$

$$\text{But } m \neq n \text{ (given)} \quad \left| \quad a_{m+n} = 0 \right.$$

**Or**

Let three numbers in AP be  $a-d, a, a+d$

$$(a-d) + a + (a+d) = 18$$

$$3a = 18$$

$$a = 6$$

Numbers are  $6-d, 6, 6+d$

$$(6-d)(6+d) = 5d$$

$$\Rightarrow 36 - d^2 = 5d$$

$$\Rightarrow d^2 + 5d - 36 = 0$$

$$\Rightarrow d^2 + 9d - 4d - 36 = 0$$

$$\Rightarrow (d+9)(d-4) = 0$$

$$d = -9 \quad \text{or} \quad d = 4$$

Numbers are

$$6 - (-9), 6, 6 + (-9)$$

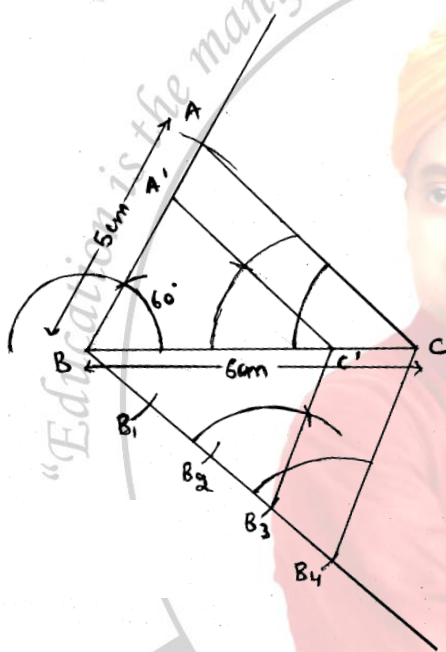
$$15, 6, -3$$

Numbers are

$$6-4, 6, 6+4$$

$$2, 6, 10$$

25.



A' BC' is the required triangle.

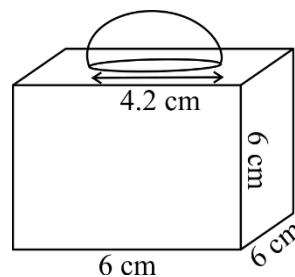
26. Total surface area of decorative block = Total surface area of cube + curved surface area of hemisphere – area of circle

$$6(6)^2 + 2\pi\left(\frac{4.2}{2}\right)^2 - \pi\left(\frac{4.2}{2}\right)^2$$

$$= 216 + \pi(2.1)^2$$

$$= 216 + \frac{22}{7} \times \frac{21^3}{10} \times \frac{21}{10}$$

$$= 216 + 13.86$$



$$= 229.86 \text{ cm}^2$$

Volume of decorative block = volume of cube + Volume of hemisphere

$$= 6^3 + \frac{2}{3} \pi \left( \frac{4.2}{2} \right)^3$$

$$= 216 + \frac{2}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1$$

$$= 235.404 \text{ cm}^2$$

Or

$$\text{Let } r_1 = 12, r_2 = 20$$

$$\text{Volume of frustum} = \frac{\pi h}{3} (r_1^2 + r_2^2 + r_1 r_2)$$

$$12308.8 = \frac{3.14h}{3} (12^2 + 20^2 + 240)$$

$$h = \frac{123088 \times 3 \times 100}{10 \times 314 \times 784}$$

$$h = 15 \text{ cm}$$

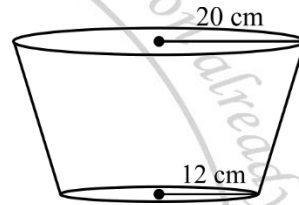
$$l = \sqrt{15^2 + (20 - 12)^2} = 17 \text{ cm}$$

Area of metal sheet = curved surface area of frustum + area of base

$$\pi l (r_1 + r_2) + \pi r_1^2$$

$$= 3.14 (17(20 + 12) + 12^2)$$

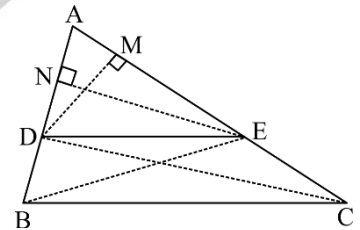
$$= 2160.32 \text{ cm}^2$$



27. Given :  $DE \parallel BC$

$$\text{To prove : } \frac{AD}{DB} = \frac{AE}{EC}$$

Join BE and CD and then draw  $DM \perp AC$  and  $EN \perp AB$



$$\text{Now, area of } \triangle ADE = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times AD \times EN$$

$$\text{So } ar(ADE) = \frac{1}{2} \times AD \times EN$$

$$\text{Similarly, } ar(BDE) = \frac{1}{2} \times DB \times EN$$

$$ar(ADE) = \frac{1}{2} \times AE \times DM$$

and  $ar(DEC) = \frac{1}{2} \times EC \times DM$

$$\therefore \frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \text{-----(i)}$$

$$\therefore \frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \text{-----(ii)}$$

$\triangle BDE$  and  $DEC$  are on the same base  $DE$  and between the same parallels  $BC$  and  $DE$ .

So  $ar(BDE) = ar(DEC)$ -----*(iii)*

$\therefore$  From (i), (ii) and (iii), we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

**OR**

Given :-  $ABC$  is a right angled triangle.

To prove :-  $AC^2 = AB^2 + BC^2$

Construction:- Draw  $BD \perp AC$

Proof:- In  $\triangle ADB$  and  $\triangle ABC$

$$\angle A = \angle A \text{ (Common)}$$

$$\angle ADB = \angle ABC \text{ (each } 90^\circ)$$

$\Rightarrow \triangle ADB \sim \triangle ABC$  (AA similarity)

So,  $\frac{AD}{AB} = \frac{AB}{AC}$  (Sides are proportional)

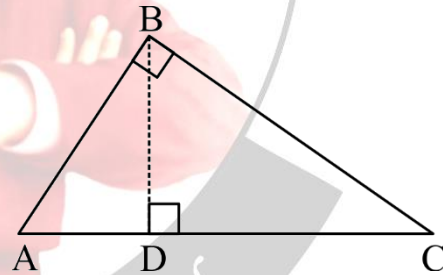
$$AD.AC = AB^2 \quad \text{(i)}$$

In  $\triangle BDC$  and  $\triangle ABC$

$$\angle C = \angle C \text{ (common)}$$

$$\angle BDC = \angle ABC \text{ (each } 90^\circ)$$

$\Rightarrow \triangle BDC \sim \triangle ABC$  (By AA)



$$\text{So, } \frac{CD}{BC} = \frac{BC}{AC}$$

$$CD.AC = BC^2 \quad (\text{ii})$$

Adding (i) and (ii)

$$AD.AC + CD.AC = AB^2 + BC^2$$

$$AC(AD + CD) = AB^2 + BC^2$$

$$AC.AC = AB^2 + BC^2$$

$$AC^2 = AB^2 + BC^2$$

Hence proved.

28.  $1 + \sin^2 \theta = 3 \sin \theta \cos \theta$

Divide both sides by  $\cos^2 \theta$

$$\Rightarrow \frac{1 + \sin^2 \theta}{\cos^2 \theta} = \frac{3 \sin \theta \cos \theta}{\cos^2 \theta}$$

$$\Rightarrow \frac{1}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = 3 \tan \theta$$

$$\Rightarrow \sec^2 \theta + \tan^2 \theta = 3 \tan \theta$$

$$\Rightarrow 1 + \tan^2 \theta + \tan^2 \theta = 3 \tan \theta$$

$$\Rightarrow 2 \tan^2 \theta - 3 \tan \theta + 1 = 0$$

$$\Rightarrow 2 \tan^2 \theta - 2 \tan \theta - \tan \theta + 1 = 0$$

$$\Rightarrow 2 \tan \theta (\tan \theta - 1) - 1 (\tan \theta - 1) = 0$$

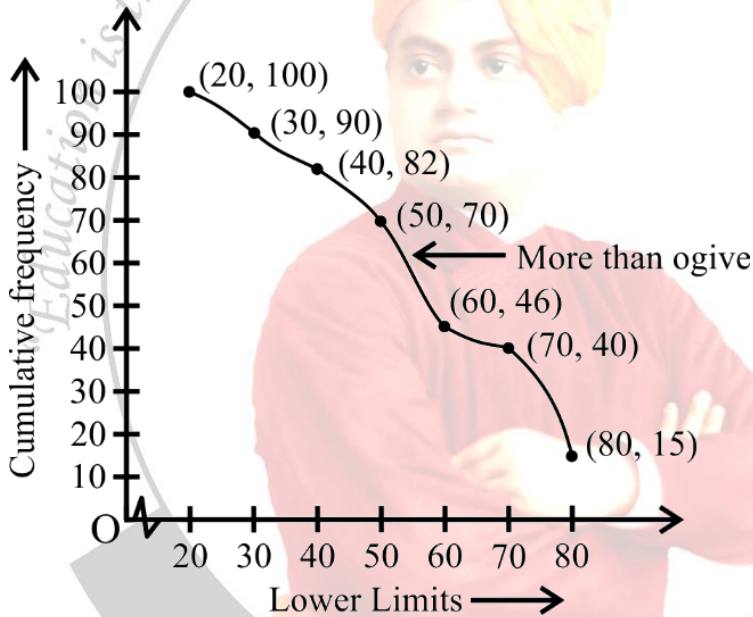
$$\Rightarrow (\tan \theta - 1) (2 \tan \theta - 1) = 0$$

$$\Rightarrow \tan \theta = 1 \quad \text{or} \quad \frac{1}{2}$$

Hence proved

29.

Observations	Cumulative frequency (cf)
More than or equal to 20	100
More than or equal to 30	90
More than or equal to 40	82
More than or equal to 50	70
More than or equal to 60	46
More than or equal to 70	40
More than or equal to 80	15



30. Let  $h$  be the height of the tower

$$CD = 40m$$

In  $\triangle ABC$

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\Rightarrow \frac{h}{BC} = \sqrt{3}$$

$$\Rightarrow BC = \frac{h}{\sqrt{3}} \dots\dots\dots(i)$$



In  $\triangle ABD$

$$\frac{AB}{BD} = \tan 30$$

$$\Rightarrow \frac{h}{BC + 40} = \frac{1}{\sqrt{3}} \quad (\text{from (i)})$$

$$\Rightarrow \frac{h}{\frac{h}{\sqrt{3}} + 40} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{\sqrt{3}h}{h + 40\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow 3h = h + 40\sqrt{3}$$

$$\Rightarrow 2h = 40\sqrt{3}$$

$$\Rightarrow h = 20\sqrt{3}$$

$$\Rightarrow h = 20(1.732)$$

$$\Rightarrow h = 34.64 \text{ m}$$

$\therefore$  Height of the tower is 34.64 m

